Physikpraktikum für Vorgerückte

Josephson Effect

Christian Walther ‹cwalther@gmx.ch›

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1 Summary

In 1962, Brian D. Josephson theoretically showed that a contact consisting of two superconductors with a very thin tunnel barrier between them can conduct an electrical current without any potential difference (DC Josephson effect). Furthermore, if there is a potential difference, the current oscillates with a frequency that depends only on the potential difference and the ratio of physical constants e/h (AC Josephson effect). Using a self made Josephson contact, the two effects are observed, and from the AC effect measurement e/h is calculated.

2 Theory

The summarized theory found in the lab manual is not very clear and it also contains mistakes. A very good theoretical explanation is given by Feynman (R. P. Feynman, R. B. Leighton, M. Sands, The Feynman Lectures on Physics, Vol. III: Quantum mechanics, Chapter 21) and won't be repeated here. The result is that the Josephson current is given by

$$I = I_0 \sin(\delta_0 + \frac{2eU}{\hbar} t),$$

i.e. a constant current for voltage $U = 0$ and an oscillating current with frequency $\frac{2e}{\hbar} U$ for $U > 0$. The phase shift δ_0 is connected to the magnetic field around the contact.

3 Making the Josephson Contact

The Josephson contact used in this experiment consists of an oxidated niobium wire enclosed in a drop of tin-lead solder (figure 1). Niobium and solder are superconductors

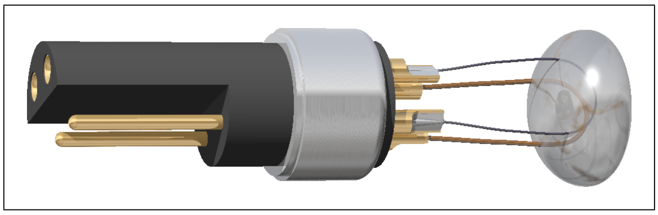


Figure 1: Josephson contact mounted on a Lemo connector. The actual contact is between the thin niobium wire and the solder droplet, the thicker copper wire is used to connect to the solder droplet. The solder is drawn translucent in order to reveal the configuration of the wires.

at liquid helium temperature (4.2 K), the oxide layer serves as the tunnel barrier. The advantage of this type of contact is that it can easily be made by hand using commonly available tools such as a soldering iron and pliers. Its disadvantage is its unreliability: most contacts don't work because the oxide layer is damaged or too thick. In my particular case, only about the fifteenth trial yielded a working contact.

The procedure of making the contact is as follows:

- A short piece of Nb wire (0.1 mm diameter) is cleaned using very fine sand paper and soldered to pins 1 and 4 of a Lemo connector. This is most easily done by cleaning one end of a long piece of wire, soldering it to the connector, cutting off the required length, cleaning the other end, bending it down and soldering it to the other pin of the connector.

- A piece of copper wire (0.2 mm diameter) is bent to a loop of about the same lenght as the Nb wire and soldered to pins 2 and 3 of the connector.

- To build the thin oxide layer on the Nb wire, it is connected to a power supply and heated by a current of 0.7 A during 1 minute. For some contacts, different durations between 30 seconds and 1.5 minutes were tried, but no clear influence on the result was found. During the heating, the wire changes its color to a dark blueish tone, sometimes also to white. Determining what color works better would require further experimentation, as the experiments were stopped after the first working contact was achieved.

- A piece of solder wire is molten to a drop and the two wire loops are dipped into it. This turned out to be a tricky task, as the solder seems to repel the oxidated Nb wire. Care has to be taken not to move the connector until the solder drop has hardened.

To test the contact, its resistance at room temperature can be measured. For a successful contact, it should be around 1 Ω. To measure the resistance, the contact is connected to the probe tube that will later be placed in the liquid helium tank, and with a constant current the voltage across the contact is compared to the voltage across a built-in 1 Ω resistor. The probe device is wired in such a way that the voltage across the contact is measured between pins 1 and 3 of the Lemo connector, whereas the current goes through pin 2 and 4. This way, negligible current goes through the voltage measurement circuit, and only the resistance of the contact itself is measured.

An easier method is measuring the resistance with an ohmmeter, though with this method, one not only measures the resistance of the contact, but additionally the resistance of the solder points of the connector, which can be a significant contribution. For a quick first test it is however accurate enough.

Most contacts at this point showed a resistance between 100 Ω and infinity. By melting the solder drop again, moving the wires around a little, and adding some fresh solder, most of them could eventually be brought into the 0.1–10 Ω range.

4 Measurement

DC Effect

The contact at the end of the probe tube is slowly lowered into a liquid helium tank to cool it down to 4.2 K. A variable current between 0 and 5 mA is then sent through pins 2 and 4 of the Lemo connector. To measure this current, the voltage drop across a 100 Ω resistor is fed to the X input of an XY recorder. The voltage measured across pins 1 and 3 of the Lemo connector is amplified by 100 and then fed to the Y input of the XY recorder. By linearly sweeping through the full current range, the I-U characteristic of the contact is recorded.

The expected result for the DC Josephson effect is that the voltage stays at zero as the current is increased, until a critical current I_{max} is reached where superconductivity breaks down and the voltage starts to grow with increasing current.

The measured result for about the first 15 contacts was that the voltage grew immediately from zero current on. Then finally one contact worked as expected. Incidentally, it was about the most horrible looking one, with very crumpled wires and lots of rosin residues – it literally fell apart right after the experiment.

AC Effect

Because the Josephson frequency $\frac{2e}{\hbar}U$ is in the GHz range for our parameters, it cannot be measured electronically. The radiation emitted by the contact could be measured, but it is very weak and difficult to access in the cryostat. Therefore, an alternative way to measure the frequency is used: The contact is placed in the microwave radiation of a Gunn diode of known frequency (measured by measuring the wavelength of a standing wave). One then observes equidistant steps in the I-U curve of the contact: over certain current ranges, the voltage takes a constant value that makes the Josephson frequency a whole multiple of the microwave frequency. This behavior is explained by the following calculations:

The microwave with angular frequency ω_1 modulates the voltage across the Josephson contact:

$$U(t) = U_0 + U_1 \cos(\omega_1 t).$$

Therefore, also the instantaneous angular frequency of the Josephson oscillation is modulated, and the argument of the sine becomes

$$\int_0^t \omega(t)dt = \int_0^t \frac{2e}{\hbar}U(t)dt = \int_0^t \Big(\underbrace{\frac{2eU_0}{\hbar}}_{\omega_0} + \frac{2eU_1}{\hbar}\cos(\omega_1 t)\Big)dt = \omega_0 t + \frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t).$$

Taking into account a possible phase shift between the Josephson oscillation and the microwave oscillation, the Josephson equation is:

$$I = I_A \sin\left(\omega_0 t + \frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t)\right) + I_B \cos\left(\omega_0 t + \frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t)\right).$$

Using the Bessel function identities

$$\sin(z\sin\theta) = 2\sum_{k=0}^{\infty} J_{2k+1}(z)\sin((2k+1)\theta) = \sum_{k\in\mathbb{Z}\text{ odd}} J_k(z)\sin(k\theta),$$

$$\cos(z\sin\theta) = J_0(z) + 2\sum_{k=1}^{\infty} J_{2k}(z)\cos(2k\theta) = \sum_{k\in\mathbb{Z}\text{ even}} J_k(z)\cos(k\theta)$$

this can be expanded:

$$\begin{aligned} I = I_A &\left(\sin(\omega_0 t)\cos\left(\frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t)\right) + \cos(\omega_0 t)\sin\left(\frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t)\right)\right) \\ +I_B &\left(\cos(\omega_0 t)\cos\left(\frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t)\right) - \sin(\omega_0 t)\sin\left(\frac{2eU_1}{\hbar\omega_1}\sin(\omega_1 t)\right)\right) \end{aligned}$$

$$
\begin{aligned}
= \quad & I_A \Bigg(\sum_{k\in\mathbb{Z}\text{ even}} J_k\left(\frac{2eU_1}{\hbar\omega_1}\right) \underbrace{\cos(k\omega_1 t)\sin(\omega_0 t)}_{\frac{1}{2}\left(\sin((k\omega_1+\omega_0)t)-\sin((k\omega_1-\omega_0)t)\right)} \\
& \quad + \sum_{k\in\mathbb{Z}\text{ odd}} J_k\left(\frac{2eU_1}{\hbar\omega_1}\right) \underbrace{\sin(k\omega_1 t)\cos(\omega_0 t)}_{\frac{1}{2}\left(\sin((k\omega_1+\omega_0)t)+\sin((k\omega_1-\omega_0)t)\right)} \Bigg) \\
& + I_B \Bigg(\sum_{k\in\mathbb{Z}\text{ even}} J_k\left(\frac{2eU_1}{\hbar\omega_1}\right) \underbrace{\cos(k\omega_1 t)\cos(\omega_0 t)}_{\frac{1}{2}\left(\cos((k\omega_1+\omega_0)t)+\cos((k\omega_1-\omega_0)t)\right)} \\
& \quad - \sum_{k\in\mathbb{Z}\text{ odd}} J_k\left(\frac{2eU_1}{\hbar\omega_1}\right) \underbrace{\sin(k\omega_1 t)\sin(\omega_0 t)}_{\frac{1}{2}\left(\cos((k\omega_1+\omega_0)t)-\cos((k\omega_1-\omega_0)t)\right)} \Bigg)
\end{aligned}
$$

$$
\begin{aligned}
= \quad & \frac{I_A}{2} \sum_{k\in\mathbb{Z}} J_k\left(\frac{2eU_1}{\hbar\omega_1}\right) \Big(\sin((k\omega_1+\omega_0)t) - (-1)^k \sin((k\omega_1-\omega_0)t) \Big) \\
& + \frac{I_B}{2} \sum_{k\in\mathbb{Z}} J_k\left(\frac{2eU_1}{\hbar\omega_1}\right) \Big(\cos((k\omega_1+\omega_0)t) + (-1)^k \cos((k\omega_1-\omega_0)t) \Big).
\end{aligned}
$$

Averaging this over time gives zero for every term, except possibly for the last for a particular k: $\cos((k\omega_1-\omega_0)t)$ can be constant ($= 1$) if $k\omega_1 - \omega_0 = 0$. This means that the Josephson current has a DC component if and only if there is a $k \in \mathbb{Z}$ such that $\omega_0 = k\omega_1$.

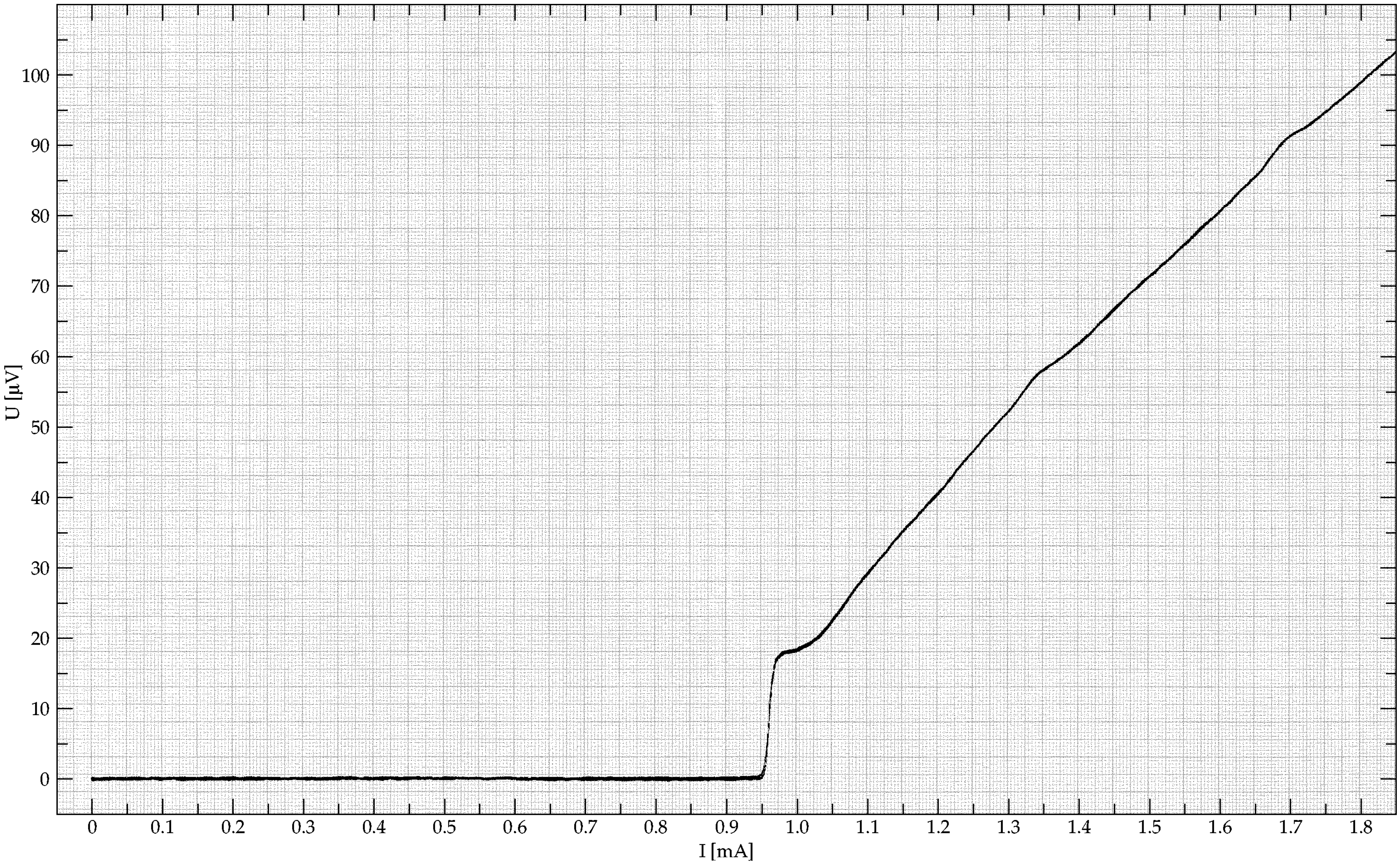
5 Results

The recorded I-U characteristics of the contacts are shown on the following two pages.

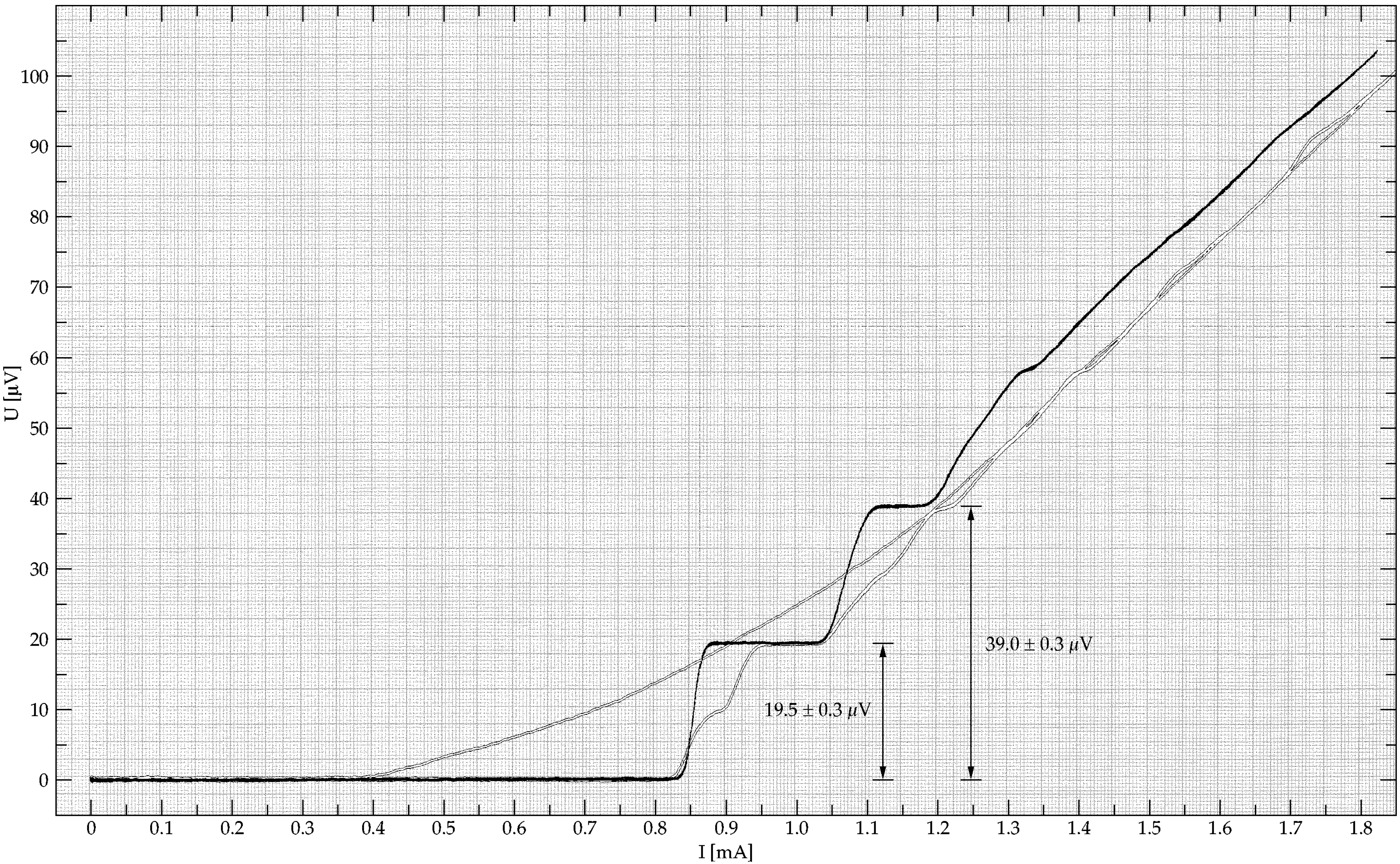
DC Effect (Page 6)
This recording looks as expected: There is current without voltage as long as $I < I_{max} \approx 0.95$ mA, for higher current superconductivity breaks down and the voltage grows. Several recordings were made on top of each other, and the curves agree almost perfectly.

AC Effect (Page 7)
Three curves with different microwave intensities were recorded. The two stepped curves again were taken several times with almost perfect agreement. The dependence on the microwave intensity however (or at least on the position of the microwave current adjustment knob, which seems to be a little loose) is so strong and irregular that it is almost impossible to reproduce one particular curve shape after having changed the microwave intensity.

The black curve shows clear steps that will be used to calculate e/h. The first one of the white curves exhibits additional small steps in the middle between the main steps, which come from the second harmonic that was not taken into account in our simplified Josephson equation. The second white curve shows an unexpected result:



U [μV]
0
10
20
30
40
50
60
70
80
90
100
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
I [mA]



U [µV]
I [mA]
19.5 ± 0.3 µV
39.0 ± 0.3 µV
0
10
20
30
40
50
60
70
80
90
100
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1.0
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8

a smoothly increasing voltage instead of a clear I_{max} and steps. Considering that no other curve of this shape could be reproduced, the most plausible explanation is an intermittent failure of the measurement apparatus.

For the AC measurement $I_{max} \approx 0.83$ mA is smaller than for the DC measurement, because the high frequency microwave field encourages tunneling of normal-conducting electrons and thus makes superconductivity break down sooner.

With an accuracy of 0.5 mm on the millimeter grid, the first two steps of the black curve are measured to be $U_1 = (19.5 \pm 0.3)\ \mu$V and $U_2 = (39.0 \pm 0.3)\ \mu$V. The 100× amplifier in the voltage measurement apparatus is specified to have a temperature drift of 0.2 μV/°C (on the output, corresponding to 100× less on the input), adding negligible error. The calibration accuracy of the XY recorder is unknown, a rough estimate of 1% increases the voltage errors to 0.4 and 0.5 μV, respectively. The microwave frequency is given as $f = 9.73$ GHz. Assuming an accuracy of 0.01 GHz, we get the following measurements for e/h:

$\frac{e}{h} = \frac{f}{2U_1}$	$\frac{e}{h} = \frac{f}{U_2}$	$\langle \frac{e}{h} \rangle$
$(2.49 \pm 0.05) \cdot 10^{14} \frac{\text{C}}{\text{Js}}$	$(2.49 \pm 0.03) \cdot 10^{14} \frac{\text{C}}{\text{Js}}$	$(2.49 \pm 0.03) \cdot 10^{14} \frac{\text{C}}{\text{Js}}$

The small difference remaining to the literature value $\frac{e}{h} = 2.418 \cdot 10^{14} \frac{\text{C}}{\text{Js}}$ is adequate to our relatively crude laboratory equipment.